Unit 15
Day 5

Rational Root theorem:

If p/q is a rational number written in lowest terms, and if p/q is a zero of f, a polynomial function with integer coefficients, then p is a factor of the constant term and q is a factor of the leading coefficient.

Corollary of the rational root theorem:

If the leading coefficient of a polynomial function with integral coefficients is 1, then any rational zeros of the function are integers and factors of the constant terms.

Consider the way we solve the following quadratic:

Notice, these coefficients must be factors of the lead coefficient of the quadratic.

$$12x^2+16x-3=0$$

$$(6x-1)(2x+3)=0$$

Notice, these integers must be factors of the constant of the quadratic.

$$6x-1=0 \qquad 2x+3=0$$

$$6x=1 \qquad 2x=-3$$

$$x=\frac{1}{6} \qquad x=-\frac{3}{2}$$

Therefore, the denominator of a rational root must be a factor of the quadratic's lead coefficient and the numerator must be a factor of the quadratics constant. This is true for polynomials of degree greater than 2. We use the rational root theorem to list the factors of p and q and then get the possible zeros by creating the ratios of those factors. Below you will see this done for the above quadratic.

factors of p	$\pm1,\quad \pm3$
factors of q	$\pm1,\quad \pm2,\quad \pm3,\quad \pm4,\quad \pm6,\quad \pm12$
p/q	$\pm1,\quad \pm\frac{1}{2},\quad \pm\frac{1}{3},\quad \pm\frac{1}{4},\quad \pm\frac{1}{6},\quad \pm\frac{1}{12},\quad \pm3,\quad \pm\frac{3}{2}\quad \pm\frac{3}{4}$

Boundedness Theorem:

Let f(x) be a polynomial of degree $n \geq 1$ with real coefficients and with a positive leading coefficient. If f(x) is divided synthetically by :

a) $k>0$ and all numbers in the result are non-negative, then f(x) has no zero greater than k.

b) $k<0$ and the numbers in the result alternate signs (0 pos. or neg as needed), then f(x) has no zero less than k.

Intermediate value theorem:

If f(x) is polynomial with only real coefficients , and if for real #'s a and b , the values of f(a) and f(b) are opposite in sign then there exists at least one zero between a and b.

Factor completely and identify the zeros.

1) $f(x)=4x^3+16x^2+19x+6$

\# of possible zeros: 3

factors of p: $\pm 1, \pm 2, \pm 3, \pm 6$

factors of q: $\pm 1, \pm 2, \pm 4$

possible rational zeros:

p/q $\pm 1, \pm 2, \pm \frac{1}{4}, \pm \frac{1}{2}, \pm 3, \pm 3/2, \pm 3/4, \pm 6$

$f(0)=6$

	4	16	19	6	
0				6	
1	4	20	39	45	UB
-1	4	12	7	-1	IV btw 0, -1
$-\frac{1}{2}$	4	14	12	0	✓

$f(x)=(x+\frac{1}{2})(4x^2+14x+12)$

$f(x)=2(x+\frac{1}{2})(2x^2+7x+6)$

$f(x)=(2x+1)(2x+3)(x+2)$

$\boxed{-\frac{1}{2}, -\frac{3}{2}, -2}$

2) $f(x)=2x^4+15x^3+28x^2-9x-36$

\# of possible zeros: 4

factors of p: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm9, \pm12, \pm18, \pm36$

factors of q: $\pm1, \pm2$

possible rational zeros:

$\pm1, \pm\frac{1}{2}, \pm2, \pm3, \pm\frac{3}{2}, \pm4, \pm6, \pm9, \pm\frac{9}{2}, \pm12, \pm18,$
±36

	2	15	28	-9	-36	
0					-36	$f(0)=-36$
1	2	17	45	36	0	UB

$f(x)=(x-1)(2x^3+17x^2+45x+36)$

	2	17	45	36	$f(0)=36$
-1	2	15	30	6	> IU
-2	2	13	19	-2	
$-\frac{3}{2}$	2	14	24	0	✓

$f(x)=(x-1)(x+\frac{3}{2})(2x^2+14x+24)$

$f(x)=(x-1)(x+\frac{3}{2})(2)(x^2+7x+12)$

$) \quad)(\;)(1 \quad 1 \quad 12)$

$f(x) = (x-1)(2x+3)(x^2+7x+12)$

$f(x) = (x-1)(2x+3)(x+3)(x+4)$

$1 \quad -3/2, \; -3, -4$